RELIABILITY FOR THE ANALYSIS OF [redacted]

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BAYESIAN PERSPECTIVE ©

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PRELIMINARIES

WHAT IS RELIABILITY?

IT IS THE QUANTIFICATION OF A CERTAIN TYPE OF UNCERTAINTY ASSOCIATED WITH THE EFFICACY AND SAFETY OF LARGE, COMPLEX SYSTEMS, TO INCLUDE BIOLOGICAL SYSTEMS WHEREIN IT GOES UNDER THE NAME OF "SURVIVAL ANALYSIS".

WHY RELIABILITY ?

IT IS ONE OF THE TWO NECESSARY INGREDIENTS NEEDED FOR MAKING LOGICAL DECISIONS (IN THE FACE OF UNCERTAINTY) CONNECTED WITH THE EFFICACY AND SAFETY OF SYSTEMS.

THE OTHER INGREDIENT IS THE UTILITY (COSTS AND/OR REWARDS) THAT OCCUR AS A CONSEQUENCE OF ANY CHOSEN DECISION. (ACTION)

WHAT DO WE MEAN BY RISK ANALYSIS ?

IT IS THE PROCESS OF ASSESSING RELIABILITIES AND UTILITIES, TO INCLUDE IDENTIFYING THE CONSEQUENCES.

WHY MUST WE QUANTIFY UNCERTAINTY

BY QUANTIFICATION, WE MEAN THE MEASUREMENT OF UNCERTAINTY, AND BY MEASUREMENT WE MEAN COMPARISON AGAINST A SCALE (EG. FEET FOR DISTANCE AND POUNDS FOR WEIGHT).

MEASUREMENT IS NECESSARY FOR INVOKING THE LOGICAL METHOD (EG. MATHEMATICS).

"WITHOUT MEASURING IT, WE CANNOT TALK ABOUT IT" - LORD KELVIN.

THUS TO QUANTIFY UNCERTAINTY, WE NEED A SCALE OF MEASUREMENT.

WHAT SCALE(S) FOR MEASURING UNCERTAINTY?

PROBABILITY IS THE OLDEST AND PERHAPS THE MOST COMMONLY USED SCALE.

SOME ALTERNATIVES ARE: POSSIBILITY, BELIEF, PLAUSIBILITY, FUZZY MEASURES, CONFIDENCE INTERVALS, ETC. V POINT ESTIMATES

WHAT ARE THE STRENGTHS OF PROBABILITY:

1) IT HAS A FOUNDATION THAT IS FIRMLY GROUNDED IN COHERENT BETTING AND AXIOMS OF COHERENT BEHAVIOUR.

2) ITS CALCULUS LEADS US TO A PRESCRIPTION FOR DECISION MAKING UNDER UNCERTAINTY.

WHY THEN ALTERNATIVES TO PROBABILITY?

THE AXIDMATIZATION OF PROBABILITY (BY KOLMOGOROV) IS BASED ON A STRUCTURE (NAMELY, PRECISE CLASSIFICATION, AND A ONE-TO-MANY MAPPING) THAT SOME FIND UNSATISFACTORY, AND THUS SEEK (LIMITING) ALTERNATIVES.

HOWEVER, TO THE BEST OF MY KNOWLEDGE, THE ALTERNATIVES DO NOT HAVE A BEHAVIOURISTIC FOUNDATION AND DO NOT LEAD TO A PRESCRIPTION FOR MAKING DECISIONS.

ALSO, SOME ALTERNATIVES MAY LEAD TO INADMISSIBILITY.

SOME QUALIFYING COMMENTS

1. AS A WORD OF CAUTION, THE [illegible] OF COHERENT BEHAVIOUR, UPON WHICH PROBABILITY & ITS CALCULUS ARE BASED ARE SAID TO BE NORMATIVE (I.E. THEY TELL US HOW TO BEHAVE). IN ACTUAL PRACTICE, INDIVIDUALS MAY NOT BEHAVE ACCORDING TO THE DICTATES OF NORMATIVE BEHAVIOUR.

2. RECENT WORK (WITH JANE BOOKER) ARGUES THAT IMPRECISE CLASSIFICATION NEED NOT BE A REASON FOR SEEKING ALTERNATIVES TO PROBABILITY

WHAT IS BAYESIAN INFERENCE?

WHEN THE QUANTIFICATION OF UNCERTAINTY IS SOLELY BASED ON PROBABILITY & ITS CALCULUS, INFERENCE IS SAID TO BE BAYESIAN.

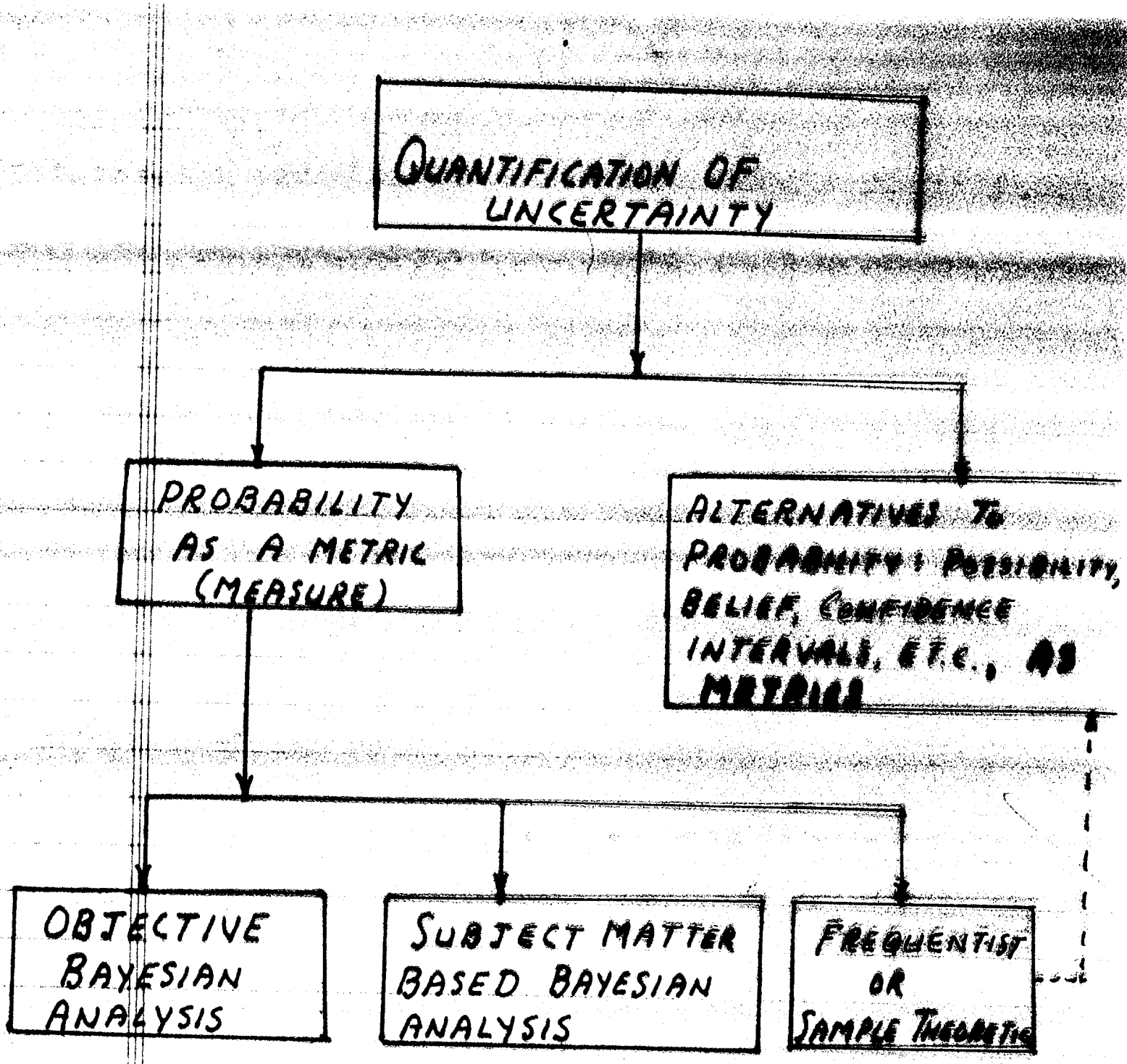
IN OTHER WORDS, A BAYESIAN IS STRICT IN HIS/HER ADHERENCE TO THE RULES OF PROBABILITY.

WITHIN THE CLASS OF BAYESIANS THERE ARE TWO TYPES:

i) OBJECTIVISTS (JEFFREYS, JAYENS, LAPLACE)

ii) SUBJECT MATTER* SPECIALISTS (DE FINETTI, SAVAGE, LINDLEY)

* (SCIENCE, ECONOMICS, ENGINEERING)



SCHEMATA SHOWING BAYESIAN, FREQUENTIST, AND NON-PROBABILISTIC INFERENTIAL PARADIGMS.

A SIMPLE EXAMPLE ILLUSTRATING THE ROLE OF PROBABILITY AND UTILITY IN DECISION MAKING UNDER UNCERTAINTY

PROBLEM: SHOULD WE OUTFIT A NEWLY DESIGNED AIRPLANE WITH ONE OR WITH TWO ENGINES?

RISK ANALYSIS:

- LET C_1 BE THE COST OF ACQUIRING & INSTALLING AN ENGINE.
- LET C_2 BE THE LOSS INCURRED DUE TO AN AIRCRAFT FAILURE; $C_2 >> C_1$.
- LET C BE THE REWARD RECEIVED UPON A SUCCESSFUL FLIGHT.

- LET p_1 BE THE PROBABILITY OF FAILURE OF AN ENGINE DURING ITS MISSION.

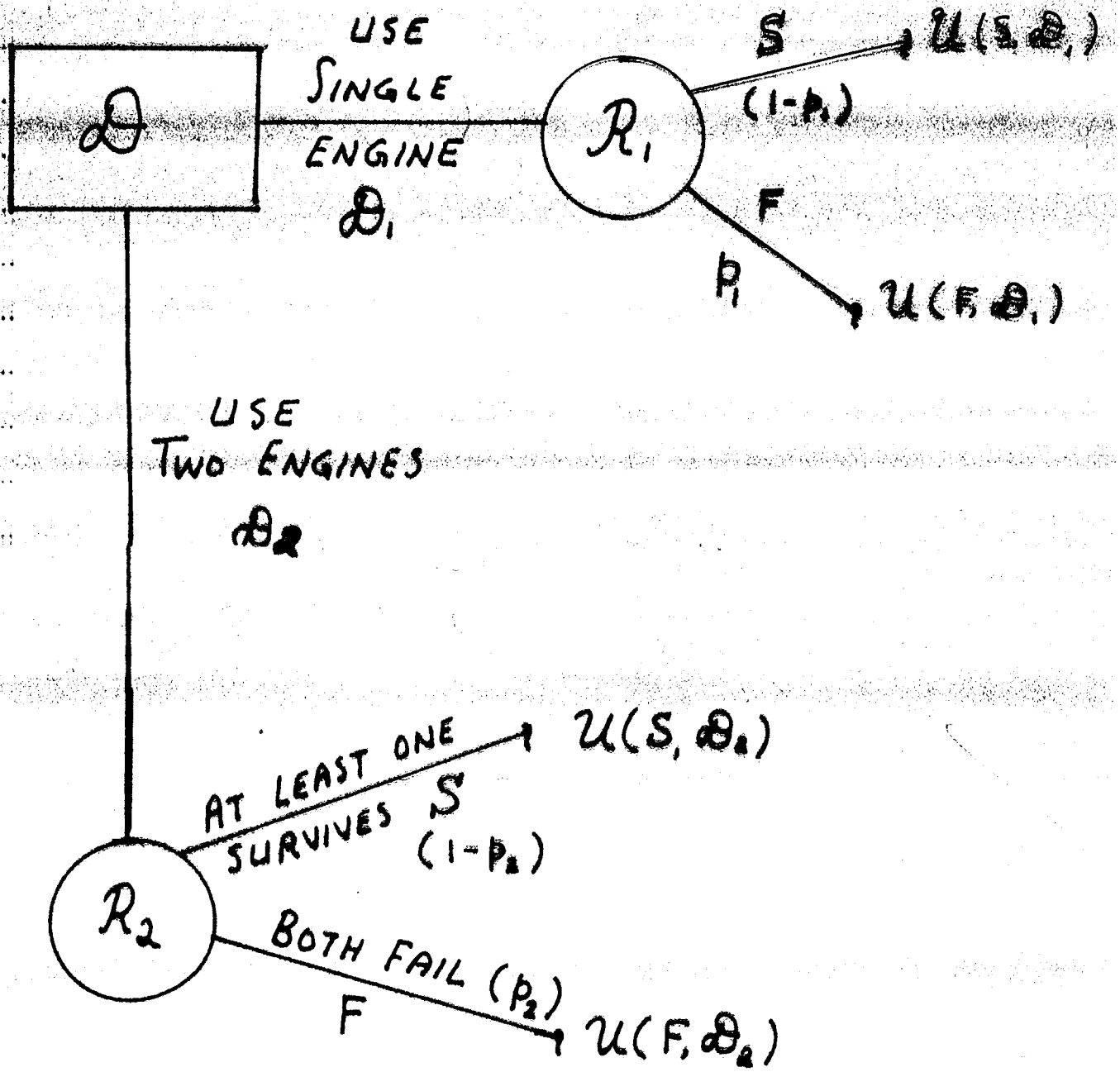
- LET p_2 BE THE PROBABILITY OF FAILURE OF BOTH ENGINES DURING THEIR MISSION. (OFTEN, $p_2 = p_1 \cdot p_1$).

THERE ARE TWO CONSEQUENCES TO EACH DECISION WE MAY MAKE.

MISSION (FLIGHT) SUCCESS - S, OR

MISSION FAILURE - F.

WE START BY CONSTRUCTING A DECISION TREE



DECISION TREE FOR AIRPLANE DESIGN PROBLEM.

SOLUTION

AT R_1 WE COMPUTE THE EXPECTED UTILITY OF DECISION α_1, AS:

$$(1-p_1) * u(S, \alpha_1) + p_1 \cdot u(F, \alpha_1).$$

WHERE

$u(S, \alpha_1) = (C - C_1)$ AND

$u(F, \alpha_1) = (0 - C_1)$.

SIMILARLY AT R_2 WE HAVE:

$$(1-p_2) * u(S, \alpha_2) + p_2 * u(F, \alpha_2)$$

WHERE

$u(S, \alpha_2) = (C - 2C_1)$, AND

$u(F, \alpha_2) = (0 - 2C_1)$.

THUS THE EXPECTED UTILITY OF DECISION $\mathcal{D}_1$ IS:

$$(1-p_1) * (c - c_1) + p_1 * (0 - c_1) = c(1-p_1) - c_1$$

AND THE EXPECTED UTILITY OF DECISION $\mathcal{D}_2$ IS:

$$(1-p_2) * (c - 2c_1) + p_2 * (0 - 2c_1) = c(1-p_2) - 2c_1.$$

WE CHOOSE, BY THE PRINCIPLE OF MAXIMIZATION OF EXPECTED UTILITY (MEU), DECISION $\mathcal{D}_2$ (USE 2 ENGINES) IF

$$c(1-p_2) - 2c_1 > c(1-p_1) - c_1, \text{ OR IF}$$

$$c(p_1 - p_2) > c_1.$$

OTHERWISE, WE CHOOSE DECISION $\mathcal{D}_1$ (USE SINGLE ENGINE)

IF $p_2 = p_1^2$, THEN, THE ABOVE BOILS DOWN TO CHOOSING δ_2 WHEN

$$p_1(1-p_1) > \frac{c_1}{c};$$

OTHERWISE, WE CHOOSE δ_1.

COMMENTARY.

1. THE ROLE OF PROBABILITIES AND UTILITIES IN MAKING DECISIONS IS CLEAR.

2. IT IS THE CALCULUS OF PROBABILITY THAT LEADS US TO THE MAXIMIZATION OF EXPECTED UTILITY, AS A PRESCRIPTION FOR TAKING ACTION.

ALTERNATIVES TO PROBABILITY NEED TO PROVIDE A SIMILAR PRESCRIPTION.

THE ABOVE, PLUS THE FACT THAT THE CALCULUS OF PROBABILITY HAS AN AXIOMATIC FOUNDATION THAT IS GROUNDED IN COHERENT BETTING, AND BEHAVIOURISTIC RULES, IS THE STRONGEST ARGUMENT IN FAVOUR OF THE BAYESIAN PARADIGM.

THE FACT THAT THE BAYESIAN RECIPE CAN ADDRESS PROBLEMS (LIKE ONE-OF-A-KIND SYSTEMS, INFORMATION SAVINGS ON SAMPLE SIZE, FUSION, PREDICTION, ETC.) IS SIMPLY A DESIRABLE AND ATTRACTIVE BY-PRODUCT.

THUS THE KEY ARGUMENT FOR ADVOCATING A BAYESIAN POINT OF VIEW IS PHILOSOPHIC AND MATHEMATICAL.

BUT THIS PHILOSOPHIC DISPOSITION ALSO ENTAILS A PRICE TO BE PAID. IT TAKES THE FORM OF TWO ISSUES:

1. ACTUAL BEHAVIOUR OF HUMANS IS NOT ALWAYS NORMATIVE

2. SPECIFICATION OF THE PRIOR ENTAILS DIFFICULTIES (BECAUSE NOW ONE NEEDS TO UDERSTAND THE UNDERLYING SCIENCE AND ENGINEERING OR ECONOMIC THEORY) AND PRIORS NEED NOT BE UNIQUE.

THE BAYESIAN'S RESPONSES TO THE ABOVE TWO ISSUES TAKE THE FOLLOWING FORMS:

1. THE BEHAVIOURISTIC AXIOMS DICTATE HOW ONE OUGHT TO BEHAVE. THEY ARE A PRESCRIPTION FOR RATIONAL BEHAVIOUR.

2. ONE HAS NO BUSINESS WORKING ON A PROBLEM THAT ONE DOES NOT UNDERSTAND. THUS STUDYING THE UNDERLYING SCIENCE AND ENGINEERING IS A DESIRABLE THING TO DO. (WEATHER YOU ARE TESTING AN ENGINEERING UNIT FOR SUCCESS OR FAILURE SHOULD NOT BE VIEWED IN THE SAME VIEN AS STUDYING THE SEX OF NEWBORN BABIES).

3. NON-BAYESIAN METHODS SOMETIMES LEAD TO INFERENCES THAT ARE <u>INADMISSIBLE</u>. THIS IS A HEAVY PRICE TO PAY.

EXAMPLES OF INADMISSIBILITY

1. ESTIMATES OF FAILURE RATES (AND DENSITIES) THAT ARE NEGATIVE!

2. CONFIDENCE LIMITS OF THE TYPE $(-\infty, +\infty)$ WHOSE PROBABILITY OF COVERAGE IS $(1-\alpha)$ FOR $\alpha > 0$! (THE FEILLER PROBLEM)

3. DECISION PROCEDURES THAT CAN BE MANIPULATED LEGALLY TO OBTAIN A DESIRED DECISION (MIL-STD 781C SEQUENTIAL LIFE TESTING FOR THE EXPONENTIAL.

THUS AS A MATTER OF PRINCIPAL SOME DO NOT USE PROCEDURES THAT COULD LEAD TO A TRAP, EVEN IF THE ALTERNATE PROCEDURES DEMAND MORE OF THE USER, SUCH AS SPECIFYING A PRIOR.

INDEED THERE IS NO KNOWN SITUATION WHEREIN THE USE OF A BAYESIAN APPROACH HAS RESULTED IN AN INADMISSIBLE SOLUTION OR EVEN AN INCONSISTENT ESTIMATE. (IN OTHER WORDS THE BAYESIAN SOLUTION IS A SAFE BET ONCE A PRIOR HAS BEEN AGREED UPON.

ELICITING PRIORS IS NOW THE MAIN JOB OF A STATISTICIAN.