

## **Enclosure B - Statistical Process Control for process control of microbiological levels**

### **Executive Summary**

As mentioned in Appendix D, the purpose of sampling and thus measuring something is to make some type of inference or evaluation of some population property. In Appendix D a short discussion of the effects of sample and measurement errors on evaluation was given. This appendix discusses in more detail one general sampling application: Statistical Process control (SPC), which is a type of quality control (QC) sampling used to control a process. Statistical Process Control (SPC) has been used in the manufacturing setting for many years for controlling the quality of produced items. Recently its applications have been extended to microbiological output for use, successfully we believe, in ensuring the safety of processed foods or other items that might present a hazard to consumers of the product. In addition SPC can be used by laboratories in helping ensure that the measurement process is “in control” – that is, that the measurement deviations from the true value, over time can be considered as being independent of time and within specifications that might have been determined from collaborative or inter-laboratory studies. This can be accomplished using split samples or check samples, and occasionally comparison with another more authoritative method.

There are two features that characterize SPC and differentiate it from other types of sampling, namely acceptance and survey sampling. These types of sampling involve taking samples from a well defined population of units, specifying an upper bound to the number of samples that would be taken, and, from the results obtained from these samples, making a decision or an evaluation about the population that was sampled. As opposed to these types of sampling, SPC sampling does not involve specifying a fixed upper bound number of samples or necessarily identifying clearly a population of units. Rather, SPC involves sequential sampling over time, accompanied by a set of rules or criteria that are used to make decisions or evaluate, not so much a well defined set of units, but rather the process that is creating the units. The second feature that characterizes SPC is that the underlying values of parameters that are used to construct the rules are derived from results from sample units that were created by the process itself. In order for this to be done in meaningful way, the parameter values should be reflecting the process when it is in control. Thus, SPC as a subject matter, involves methodology for judging this – when can it be considered that a process is in control so that the rules that are to be used for evaluating whether or not the process is or remains in control are valid. SPC involves evaluation of the process and not specifically whether produced units or obtained measurements are within some pre-defined specifications.

Laboratories can use QC procedures for assuring that the measured results being produced are within specifications that are defined by repeatability or reproducibility parameters. SPC though offers a degree of flexibility that takes into account the actual system or process of measurement, insofar as the criteria for evaluation are not derived from outside the process but are derived from within the process itself. The full application of SPC entails a continuous examination of the data with the purpose of not

only just judging whether or not a process is not producing as it should, but also that the process has the capability of producing better than it was initially thought it should, by helping identify areas of potential improvement. That is, evaluative criteria can change, taking into account the potential capability of the process.

Thus, this document has a twofold purpose. The first purpose, the primary one and the reason for the document, is to present “performance standards” regarding the application of SPC for microbiological output of a process. However, a second purpose is to provide a simple introductory paper that could serve as a beginning point for learning about SPC and its application for microbiological data. Thus examples in Appendix G1 are given that demonstrate principles that are rooted in the performance standards.

The sampling work group is recommending the following “performance standards” with respect to implementing SPC for microbiological data. The performance standards are not meant to prescribe procedures or criteria that should be used for evaluating processes; rather they are meant to provide guidance and a methodology to be used for developing a SPC sampling plan. Following the performance standards are discussions of them, a conclusion section, and specific examples (seven in all) given as Appendices of the report (BPMM report Appendix F.1). The examples include SPC for qualitative or attribute data, including binomial Poisson –like, and negative binomial distributions; continuous variable data of high levels of generic E. coli; and an example which uses SPC for tracking the occurrence of infrequent events such as the finding of E. coli O157:H7 on samples. Hopefully these examples will serve as useful material.

**Performance standards:**

1. Charts of plots of the output data over time are not only valuable for verifying calculations and having a visual picture of the variation exhibited by the process output, but also it is an integral tool to be used for identifying sources of unexpected variation in output leading to their elimination. Thus charting is a necessary tool needed to gain the full benefit of doing SPC.
2. Results to be plotted in a control chart, when the process is under control, used for statistical process control should be normal or nearly normally distributed. In cases where this is not true and an alternative known distribution cannot be assumed such as a Poisson, binomial, or negative binomial distributions<sup>1</sup>, transformations such as the log transformation for microbiological counts, arcsine transformations for binomial data, or a square root transformation for data distributed nearly as a Poisson distribution should be considered.
3. During some “initial” period of time, it is assumed that the process is operating in a relatively stable manner – or is in control. During this period the distribution of the measurements should be estimated and rules for evaluating the process should be formulated. The statistical “rule of thumb” of using about 20-30 results or more for computing means and standard deviations or other summary statistics

needed to estimate the distribution of results and construct control limits is a recommended and desirable goal.

4. Rules for evaluating process control should be set with aids assessing the two types of errors: Type 1, declaring the process out of control when it is not, and Type 2, not declaring a process out of control when it is. Typically there are two measures, depending upon the nature of the rule, that are used for assessing these errors: 1) the probabilities of the two types of errors at a given time (referred to as  $\alpha$ - and  $\beta$ - probabilities, respectively); and 2) the average run length (ARL) – the expected number of samples before an out of control signal (one of the rules being not met) is seen.
5. When a process is thought to be “in control,” the limits for assessing individual results are set at some distance from the average, expressed as standard deviation units from the mean or process target value. The default distance is 3 standard deviations<sup>ii</sup>. Limits other than these should be implemented when taking into consideration economic and public health costs of incorrect decisions regarding whether the process is in control. When developing rules, the  $\alpha$ -probability (for the Type 1 error) should be kept low, for example, below 1%.
6. There are numerous run/trend rules that can be used, such as runs test, moving averages and CUSUMS, for detecting shifts in the process mean; and rules for detecting shifts in the process variation or other auto-correlated patterns that could be due to systematic source of variation. The use of any of these may depend upon particular expected conditions that arise when the process is out of control, and the sensitivity desired for detecting such conditions. In assessing the use of these rules, one should consider the ARL. It is recommended, when the process is in control, that an ARL should exceed 100 (corresponding to a less than a 1%  $\alpha$  - error).
7. Specification Limits are not Statistical Process Control limits; specifications are either customer, engineering, or regulatory related. Statistical Process Control limits are process related. Specification limits should not be placed on a control chart insofar as these might be considered as process goals thus influencing the efficacy of SPC procedures for ensuring a controlled process, and thereby undermining the safety of the product.

## **Performance standard 1 – the necessity of charting**

Statistical process control (SPC) involves two aspects: use output data from a process to establish an expected distribution of values of some variable which is used for judging the control-status of a process when the process is (thought to be) in control; and a set of rules or criteria for which (future) output values from the process must satisfy in order not to declare, or declare presumptively, the process is out of control. In establishing the distribution to be used for determining the control status of the process, besides the output data, various other, regulative, type judgments are used that can affect the assumed distribution and the rules that are used for evaluating the process.

One feature that is included in the SPC methodology is charting – plotting of output process data values that are used for evaluating the process versus time or sample number, and examining the charted or plotted data. A question might arise is: why is this charting necessary? The implication of the question is that it may not be necessary, particular so with today’s computer technology – all that is needed is to somehow feed the data into a computer program and the program would make the calculations, determine whether or not the rules were violated and thus provide the control-status of the process. Various answers to this question can be given. One answer could be that charting provides a confirmation of the calculations; however, with today’s computer technology there are many other ways of ensuring that the calculations are correct to the extent that if there was a noted discrepancy between the plotted data and the computed results it more likely would be due to an error in plotting rather than in calculations. Thus, the answer to the question involving “looking” at a chart for the purposes of confirmation does not provide a good reason for the necessity of charting. Another answer might be based on psychology – the chart provides management with a visual picture of what is happening and this would give them a greater understanding of the process than what could be gained by examining sets of numbers and adherence of them to a set of rules. This answer by itself though would not provide a necessary reason for charting, at least not one in which a requirement of charting is recommended since there really would not appear to be a concrete gain from plotting.

However, this last answer is getting closer to the reason that compelled us to recommend, necessarily, charting, rather than just pointing out that charting is useful for the above stated reasons. The “seeing” of the chart can convey an understanding of the process that adherence to a set of rules cannot. Thus while the “looking” at charts can provide the confirmatory and psychological assurance, the “seeing” – meaning, a more in depth examination of the charted data - can provide additional information about certain aspects of the process that might have been unanticipated initially so that prior “rules” reflecting these aspects were not constructed. From “seeing” a chart, new insights might be gained that could show the inadequacy of the selected rules or could provide motivation for the development of new rules that lead to identifying unanticipated sources of error and an improvement of the process; on the other hand, however, it could lead to explorations that do not lead to improvements and thus could lead to an inefficient use of time and resources. Thus, to help prevent incorrect decisions statistical analysis (retrospectively) of data should be performed (See Appendix 2). The “look and see”

approach to charting is emphasized in SPC, notwithstanding possible pitfalls associated with this.

## **Performance standard 2 – The control distribution**

Statistical Process Control, (SPC) has been used successfully to control quality and costs of manufactured products since the late 1920's. This statistical tracking system used for monitoring processes performance was developed by Dr. Walter Shewhart<sup>iii</sup>. He discovered that variation observed in manufacturing output was visually “different” from the variation that he would expect to see for similar type characteristics in nature for a stable system. Dr. Shewhart speculated that the variation that was not expected was due to processing errors by either labor or management. In other words, if the process was “under control,” the deviations from a mean value of statistical measurements that “track” some feature or output of the process would be distributed in a “random” looking fashion without any clear patterns, “unimodally” or at least displaying some degree of “regularity” or “stability” with very few outlier values. Further, it was assumed that the errors would be symmetrically, or nearly symmetrically, distributed around the mean value. In other words, normality, or near normality, is a natural distribution to assume when a process is under control since it is then assumed that the deviations are “caused” by many, inherently uncontrolled factors, each contributing only a small amount to the magnitude of the deviation. Historically then, in the manufacturing setting, rules or control limits for assessing a process to be out of control were set symmetrically with respect to the mean value – the assumption being that a result could be equally likely above as below the mean value. Thus, the distribution of the plotted values for the control chart was assumed to be normal and the operating characteristics of the rules - the probability of declaring the process out of control as a function of the true process mean - were evaluated assuming the underlying distribution of results is the normal distribution.

For microbiological data the above assumptions may not be true – rather, often (explicit examples are given in Appendix F.1) distributions seen will not be symmetric. If the non-symmetric distribution is known, then it is possible to use this distribution directly with the accompanying mathematical calculations to derive control limits with certain desirable operating characteristics. In such a situation parameters of these distributions can be estimated by maximum likelihood estimation or other statistical procedures and control plans can be determined directly using estimated distribution. However, often these specialized assumptions cannot be made, since with processing and measurement there would be expected unavoidable differences over time that could be caused by factors related to slight variations of equipment settings, environmental conditions and personnel that cannot easily be controlled or completely eliminated. For example, it might be assumed that under ideal conditions, the plate count distribution would be Poisson, with a parameter,  $\lambda$  - representing, in this case, the expected value. But value of this parameter may not be constant from day to day, or sample to sample, rather,  $\lambda$  itself would be a random variable, taking on possibly different values for different samples. Because of this ( $\lambda$  being a random variable), the total variation seen in the obtained results would not be expected to be equal to the expected variation of results seen from a Poisson distribution. The distribution of the results thus might be represented

well as a mixture of Poisson distributions. One such distribution is the negative binomial distribution, which has two parameters.

In general though, the expected distribution when the process is in control may not be known other than it most likely would not be symmetric. And for the classical SPC control procedures (as described below for Performance Standard 3), the limits are set using sample mean and standard deviation values for results on sample collected from a process assumed to be in control or nearly so, as if the distribution of these results were generated from a nearly normal distribution. If the distribution of results is not nearly symmetric, then transformations of the output variable, for example, taking the logarithm of microbial plate counts, may induce a more symmetric looking distribution. There is often another advantage of using the transformed variable: namely, the expected standard deviation would be less dependent on the expected mean value of the particular result. Thus, if plate counts were thought to be distributed as nearly lognormal, then a log transformation would make the distribution nearly normal and the variances of each transformed result would be nearly uniform for the data. Similarly if the data results were thought to Poisson-like distributed, a square root transformation of the results would make the results more symmetrical and make the variance more uniform (Appendix 3); for the binomial distribution, the arcsine transformation,  $\sin^{-1}[(x/N)^{1/2}]$ ; and for the negative binomial, the inverse hyperbolic sine transformation,  $N^{1/2}\sinh^{-1}[(x/N)^{1/2}]$  would make the distribution more symmetric and the variance more uniform (Johnson and Kotz, 1969).

While a normal distribution of the deviations from the mean value is not an absolute necessity for applying the control techniques discussed in this paper, historically the stated probabilities describing the operating characteristics of the control plan are computed assuming normal distributions and used for motivating decision rules. As a result of these considerations, performance standard 2 is recommended.

### **Performance standards 3 and 4 – Establishing the control distribution and rules for process evaluation**

SPC is applied as follows:

- 1) During some “initial” period of time, it is presumed that the process is operating in a relatively stable manner, as described in the preceding paragraphs. This is a very important presumption and in actuality to reach this point when the process controls and parameter values are set, it may be needed an extended period of experimentation or trials. Whenever possible, independent validation of the presumption of process control should be made by other means, different from the statistical process control planning to be used, such as, for laboratory QC, the use of reference standards or cultures with known characteristics. If the distribution of results is expected to be nearly normal, then during this period statistical measurements should be distributed randomly around a mean value,  $\mu$  with a standard deviation,  $\sigma$ . Values for these parameters are estimated during this time.

- 2) Over time, the statistical measurements are plotted on a graph, called a Shewhart chart (see Appendices for examples), showing the distribution of the statistical measurements. The Shewhart chart is basically the plot of the measured values versus sample number, starting with some sample labeled 1.
- 3) If the plotted statistical measurements do not meet any one of a set of criteria the process is considered to be “out of control.”

The criteria are chosen to reflect different manifestations of “out of control” of interest to the producer. Particular types of “out of control” signals are: a) “short term” non-systematic errors that might occur that result in an unacceptable product for a given day or lot; b) persistent errors that cause a systematic deviation from the pre-designated target value,  $\mu$ ; and c) persistent errors that cause an increase in variability ( $\sigma$ ) of process output.

Decision errors in regard to deciding whether or not a process was under control are similar to decision errors guarded against by the use of statistical procedures when testing two competing hypotheses in science. That is, a Type 1 error is made by deciding that the process is out of control when, in fact the process is in control and thus would not require adjusting; and a Type 2 error occurs when a process is not adjusted (actually is out of control) but it is decided that it is not out of control and the process is left as is. The probabilities of these errors are, respectively, referred to as  $\alpha$  - and  $\beta$ - probabilities. Both of these errors could contribute to processing inefficiencies.

Processes can be affected by Type 1 and 2 errors because management and hourly workers often make adjustments that should not be made or fail to make adjustments that should be made in the attempt to “improve” the process output. The psychological forces that lead to changes or no-changes and thus errors influence the output of a process. A belief could develop, particularly the more one gains experience with the process, that ad-hoc adjustments based on one’s expert judgment would lead to a better process and output than just relying on pre-set rules as implied by charting and SPC. While in certain circumstances this may be true, often times it would not be so, and such a belief (of the advantages of following expert judgment) is not a reason to resist placing control charts on a process and using SPC. If nothing else, the use of control charts and SPC helps establish objective criteria for making adjustments (once the limits are established).

In other words, SPC and the use of rules for evaluating the process, determining  $\alpha$  - or  $\beta$ -probabilities of the rules are not meant to eliminate expert judgment; rather these activities should be viewed as an aid for making judgments helping to prevent unwarranted actions that lead to a Type 1 or Type 2 error. “Out of control” signals can be considered “presumptive” regarding whether the process is out of control; and the examination of the data once plotted can lead to judgments of “an out of control process” that the charting “rules” have not reflected. Thus performance standards 3 and 4 we consider to be necessary for preventing the dominance of expert judgment in the evaluation of a process, but is not meant to eliminate it.

Consequently, SPC in its fullest sense involves preliminary analyses or testing of the process to a point where process parameters have been determined, such that it is believed that when the process is operating in accordance with the parameter specifications, the distribution of the measured output that is being used for evaluating process control would have the characteristics described above (random, stable, nearly symmetric or with some other designated distribution). In the developmental stage of the process this assumption may not be true. The SPC and plotting techniques described here can also be used in the developmental stages; however, in this situation the criteria for out of control may need to be changed.

### **Performance standard 5 – Control limits for individual values**

Dr. Shewhart understood that in order for a control system to work effectively the rules or criteria used for determining the control-status of a process should meet a couple of requirements. These requirements include:

1. The  $\alpha$ -probabilities (of incorrectly saying a process was out of control when it was not) must be low enough so to not unnecessarily create delays in processing (which could be costly) and fatigue workers and management from looking for causes of variation that do not exist;
2. The criteria must be robust enough so that a number of probability distributions can be accommodated by the procedures; and
3. The criteria should be simple and easily “seen” on a graph<sup>iv</sup>.

As a consequence of the above considerations, Dr. Shewhart settled on his most well-known criterion that placed, what he termed, “Control Limits” a distance of three standard deviations from the process average; that is, control limits were set such that if a single measured value, (labeled often as  $X_i$ ), was either greater than  $\mu + 3\sigma$  or less than  $\mu - 3\sigma$ , with  $\mu$  being the process average or intended process target then the process was to be presumed out of control. When the underlying distribution is normal, then the probability of exceeding one of the limits is 0.135%, so that the two-sided  $\alpha$ -error is 0.27%. For most distributions expected for processes under control, the likelihood of seeing measured observations that do not satisfy these criteria is small<sup>v</sup> thus satisfying requirements 1 and 2 above. Also, the third requirement is clearly met because the limits are just horizontal lines on the chart, and it can be easily seen if a plotted point is not between the two lines, indicating “out of control.” This criterion would “catch” a processing error that might not be systematic, and, when not met, would imply that there is some aspect of processing that might not be controlled.

## Performance standard 6

### Tracking trends or shifts in the process mean value

There are many ways to evaluate “systematic” errors that would cause the mean value of the process to change. One very simple way, which can be easily seen on a Shewhart chart, is to use “run” tests, for example, to declare a process “out of control” when 8 consecutive points fall on the same side of the target value (e.g. process mean) - a run of length 8. When the process mean value equals  $\mu$ , such a pattern is highly unlikely, (assuming here a symmetrical distribution of measured values) so that when such a pattern is seen it is likely that the process mean is not equal to  $\mu$ . A run of 8 consecutive results above or below the targeted mean value has, for those 8 results, a 2 in 256 chance of occurring (accounting for the two possibilities of 8 results above the mean or 8 results below the mean) or about a 0.8% probability,  $(2(0.5^8) = 0.0078)$ . However, with such tests, it takes one result to break the pattern. A criterion might be set as: if at least 7 of 8 consecutive results are above or below the mean value, then the process would be considered as out of control (or presumptively out of control, pending further investigation). The probability of at least 7 out of 8 observations above or below the mean value has a probability of 7% (from the Binomial probability distribution with an incidence parameter with a value of 0.5). That is to say, it would be expected that 7% of any 8 consecutive sample results to have at least 7 of the results above or below the mean value when there is for each result a 50% chance of being above or below the mean value. Thus if a criterion of at least 7 of 8 results are above or below the mean value the process would be presumed out of control, the  $\alpha$ -probability would be about 7%. For a one-sided test that is, for example, a test for which the concern is only with a process change that results in an increase of the process mean value, the  $\alpha$ -probability would be 3.5%. This percentage is usually considered too high, given the costs associated with investigating a presumptive out of control signal.

Because just one result can “break” the pattern, runs tests are not very “powerful” for detecting small or even moderate shifts (relative to the standard deviation) – that is the  $\beta$ -error may be large. For example, if the mean increased by one standard deviation unit, so that the true process mean changed to:  $\mu + \sigma$ , then the probability of an individual result being below the target value,  $\mu$ , is 16%, (84% of the values will be above the target). For 8 consecutive results, the probability of having at least one result below the target value of  $\mu$  is about 75%, so that the  $\beta$  error associated with this criterion would be 0.75, (for a single run of 8 values). The criterion for the upper control limit would not help much: there is an 83% probability that all 8 results would be below the upper control limit of  $\mu + 3\sigma$ .

For this reason, moving averages and CUSUMs are often used for monitoring processes, where a moving average is the average of the results in a group of consecutive samples and a CUSUM is a procedure that accumulates iteratively deviations from the target value. Moving averages are more difficult to compute because the samples used for computing averages are always changing and thus at any time the results over a (changing) set of samples need to be known. Also there is an issue of how many samples

to use in computing the moving average (the window length of the moving average). The CUSUM (Johnson and Leone<sup>vi</sup>, 1964; Juran 1988<sup>vii</sup>) control procedure avoids these problems and is thus simpler to compute and to design. The CUSUM value is basically updated at each sample by adding the deviation:  $X_i - \mu$ , to the previous value, where  $\mu$  is the target value, for example, the (expected) process mean<sup>viii</sup>. When CUSUM values are plotted versus sample number, evidence for a shift<sup>ix</sup> in the process mean value is easily seen when the graph of the points steadily increases or decreases. For a simple charting of CUSUM<sup>x</sup> where a control limit can be depicted such that a process out of control signal would be given when the CUSUM value exceeded the limit,  $L$ , a slight modification of the CUSUM as described above can be made, namely: computing,  $S_i = \max(0, S_{i-1} + X_i - \mu)$ , where  $S_i$  is the CUSUM value for the  $i^{\text{th}}$  sample, and  $S_0 = 0$  (or some other value for a 'quick start'). When  $S_i > L$ , this would imply that there was a positive shift in the process mean some time in the recent past. (An example of the calculations is given in Appendix 2.) A similar type CUSUM can be constructed for negative shifts of the process mean. Developing limits for these is more complicated and is out of the scope of this document, however, it is encouraged that these procedures be considered when designing control charts.

To measure the effectiveness of sampling plans and sampling criteria (or rules), another parameter, called the average run length (ARL) is used. The run length, RL, is a random variable that counts the number of samples (starting at some specific sample) before the first signal for "process out of control" is given; in other words, the number of samples until at least one of the sampling plan's criteria or rules for declaring the process out of control is obtained. By convention, the sample for which there is a signal is counted, thus all run lengths are greater than or equal to 1. Control plans and their criteria are often evaluated by characterizing the distribution of the run lengths, and in particular by the expected number of samples – the average run length (ARL) - before the first "out of process" signal given, starting from some specific sample. When a process is in control, the desire is that the ARL should be large; and of course, when the process is out of control the desire is to have a small ARL. ARL has become a traditional parameter to consider, but other parameters of the run length distribution could be considered as well, for example, selected percentiles of the RL distribution.

A simple example, presented below, shows comparisons between a CUSUM rule and the "eight in a row" rule described above. In the example, a CUSUM rule is given that has a comparable ARL when the process is in control to that of the "eight in a row" rule discussed above. The following table gives the average and median run lengths. The values given in the table were determined from 20,000 simulations<sup>xi</sup>. The assumption for the underlying distribution is normal, with mean equal to  $\mu$  and standard deviation = 1. Control is when the mean,  $\mu = 0$ .

Table 1: Results: each entry determined from 20,000 simulations.

CSUSM calculations:  $S_k = \max(0, S_{k-1} + x)$ , where  $x$  is distributed normal with mean =  $\mu$  and standard deviation = 1, and  $S_0 = 0$ . The process in control mean = 0; when  $\mu > 0$  the process is out of control. Out of control signal when  $S_k > 21.5$ . The parameter “mu” is the number of standard deviation the process has drifted from the target ( $\mu$ ).

mu	CUSUM	CUSUM	8 in row	8 in row
	mean	median	mean	median
	ARL	ARL	ARL	ARL
0.0	510.7	392	506.9	351
0.1	177.1	152	299.9	210
0.2	100.8	91	183.0	129
0.3	69.6	65	121.2	86
0.4	53.8	51	82.6	59
0.5	43.2	41	58.9	43
0.6	36.5	35	43.7	32
0.7	31.4	30	33.7	25
0.8	27.5	27	26.8	20
0.9	24.7	24	22.2	17
1.0	22.2	22	18.9	15
1.1	20.3	20	16.4	13
1.2	18.6	18	14.4	11
1.3	17.2	17	13.0	10
1.4	16.0	16	11.9	8
1.5	15.0	15	11.1	8

The chart shows that for a value of  $\mu$  less than 0.7 standard deviation units but greater than zero, the CUSUM has smaller ARL than that for the “8 in a row rule.” For shifts in the mean value of  $\frac{1}{4}$  standard deviation units, the ARL for the CUSUM is nearly  $\frac{1}{2}$  that for the “8 in a row” rule.

For the above control plans, when the process is in control, the ARL is about 500 corresponding, in a sense, to an  $\alpha$  - probability of 0.2% (since for a rule on individual results with this probability of a signal, the expected value of the number of samples before the first signal using a geometric distribution would be about 500). The median value for this geometric distribution is about 345, corresponding reasonably close to the median run lengths shown above for when the process is in control.

There is a great deal of literature on designing moving averages and CUSUMs, and much useful information about these can be found in the above mentioned books or even on the internet (from a reputable organization such as the US National Institute of Standards and Technology).

### Tracking process variability

The process standard deviation,  $\sigma$ , also can be tracked in many ways. A very simple criterion based on the absolute value of the differences of consecutive measured values, MR – for moving range, can be used to track the process standard deviation. This is not a very robust measure; better might be to group more results, and compute the moving standard deviations or moving ranges for the results in a group, however, these statistics are not as easily computed and plotted and may have little meaning for

processes where data are relatively rare such as microbiological data. The MR on the other hand involves only having knowledge of the most recent and second most recent results. Another option when possible is to subgroup data into discrete subgroups and to calculate the range, (high minus low observation within a subgroup). However, for data that are expensive to obtain, hard to gather and or relatively rare, charting of MR values and using them to get a visual understanding of the process has become popular.

### **Performance standard 7 – Specification limits**

Other, non-process control related values, such as, specification limits should not be placed on control charts. The reason for this is psychology. Specifications are something that all individuals who deal with a customer are accustomed to meeting. These specifications may be engineering specifications, customer requirements or regulatory critical limits, to name just a few examples. Since people are accustomed to meeting these values, and, as a consequence, specifications are given a higher priority than control limits. From a process control stand point this is not the ideal situation, insofar as the goal of process control is to achieve the best control possible for given resource constraints. Reducing variation is a particularly important goal when microbiological quantities that could represent a hazard to human health are the object of the control procedure. For example, if specification limits are “looser” than the obtainable SPC limits for a particular process and one were to make adjustments based on the specification limits rather than the control limits then adjustments would be made less often than the SPC limits would require, thus creating type 2 errors. This lack of control of a process could mask undiscovered sources of error, which if persistent could result in a product that is unsatisfactory or unsafe. In other words, a process which is not controlled and thus for which there may be unidentified sources of variation, by the mere fact of there being unidentified sources of variation not being controlled may, without the producer being aware of it, result in unsafe product. Sampling, per se, cannot be counted on for assuring to customers a product is within specifications when the process is not in control – rather only good process control can assure that. For these reasons it is advised that only process control related values be placed on the control chart.

### **Conclusion**

The SPC chart can be an important aid in identifying when and where an investigation for a cause for the process being out of control should commence. The low  $\alpha$ -probability does not imply that, when a process is in control, “out of control” signals would not occur. However, since these occurrences are not expected frequently, the occurrence of one encourages an examination of the process in search for “Assignable Causes” for each out of control signal. However, if out of control signals occur more frequently than what would be implied by the  $\alpha$ -probability, random chance –the unlucky draw- should be ruled out as a possible reason for the signals, and that there is an “assignable cause” for the excessive variation in the process output and/or one or more of the process parameters are incorrectly set. This would then call for a more rigorous review or further study of the process. If the plot of the data shows an abrupt change from consistently being in control to consistently being out of control, then it can be

concluded with high confidence that there has been an enduring failure somewhere in the process that requires immediate remediation. The plotting of the process may reveal a gradual, progress loss of control over a series of lots or production units. This pattern could result, for example, from a piece of equipment steadily becoming out of adjustment or a progressive environmental contamination resulting from an inadequate sanitation program. Another pattern could show a transitory but reoccurring or cyclical loss of control, e.g., every Monday morning. While no explicit criteria are given for detecting these types of cyclical patterns, one could use the “run rules” of 8 in a row (discussed above), e.g., if for 8 Mondays, the plotted point is above the target value, it would be suggested that for some reason results for Monday are “out of control.” The SPC plots can also document improvement in process control resulting from deliberate alterations or added mitigations. The lower levels due to the process alterations are used to establish new process standards.

When the limits for declaring a process out of control are exceeded too frequently, a producer always has the option to accept the implied non-desirable or optimal processing. Whether this option is taken depends upon ‘costs’ (technical feasibility, monetary) of fixing the problem, e.g., taking measures that would reduce either the process mean or the process variation. For example, the likelihood of a process being declared out of control with respect to some microbiological indicator variable could be reduced by increasing the heat processing temperature. However, this mitigation requires more energy consumption and may reduce sensory and nutritional quality of the food product. Reducing the variation might be accomplished by simply improving the air circulation within the oven or the one-time expense of a new oven. This mitigation would likely have an additional benefit of reducing the proportion of product that was over cooked, thereby improving the sensory and nutritional quality. This example shows a general rule: it is generally more advantageous to reduce variation first. If that is not successful, then a process step(s) may need to be redesigned to lower the entire distribution by lowering the process mean.

In the past almost eighty years the genius of Dr. Shewhart’s methods have proven themselves, and are as effective today as four score earlier. Although Dr. Shewhart used some biological examples in his book, “Economic Control of Quality of Manufactured Product, (D. Van Nostrand Company, Inc., New York, 1931),” he did not make reference to their use with regards to microbiological data. Another classic book for Quality Control that provides various statistical process control procedures is Juran, JM, 1974 Quality Control Handbook, third edition. McGraw-Hill Book Co. NY.

The brief summary presented in this document will not adequately cover the subject matter of SPC and quality control charting procedures. To aid the reader in gaining an understanding of SPC, this document includes seven examples – presented as Appendices – that cover some microbiological uses of standard SPC Charts and variations of the standard Shewhart chart which uses Shewhart’s  $\alpha$ -level for setting control limits and other out of control rules. The examples are based on computer generated simulated data, or, in one case, constructed data; the primary purpose of these examples is just to illustrate procedures and approaches for analyzing the data and

implementing SPC. From these examples, it is hoped the reader will get an idea of the uses of control charts and will be motivated to pursue the subject matter further.

There are 7 examples, all found in Appendix F.1 – SAWG SPC Appendices:

Appendix 1: Classical SPC – generic E. coli levels, treated as a variable (continuous) data.

Appendix 2: Counts, using a Poisson distribution (C- chart).

Appendix 3: Counts, not using a Poisson distribution, but rather comparing a negative binomial distribution and a square root transformation.

Appendix 4: Proportions, using a binomial distribution (NP- chart).

Appendix 5: Proportions, using a binomial distribution with different numbers of units per sample (P- chart).

Appendix 6: Counts, using a Poisson distribution, with different sample sizes (U- chart).

Appendix 7: Infrequent events, based on exponential distribution (F- chart).

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<sup>i</sup> If such distributions are assumed then goodness-of-fit statistics should be given for verification.

<sup>ii</sup> Low values for microbiological measures would not be considered as undesirable or that, necessarily, the process is out of control. Rather consistent low values could be considered as evidence of that an improvement in the process could be made.

<sup>iii</sup> Walter A. Shewhart, “Economic Control of Quality of Manufactured Product, 1931”

<sup>iv</sup> We have not seen this requirement attributed to Dr. Shewhart, but it is certainly implied by his emphasis on charting and plotting data points.

<sup>v</sup> For all unimodal distributions, likely to be seen, using these criteria, the two-sided  $\alpha$  -probability is reported to be below 5% (Vysochanskii and Petunin, 1980<sup>v</sup>). Since, primarily with microbiological data, the concern is with an out of control process leading to high values, this would imply that the one-sided  $\alpha\alpha$  - probability would be even smaller.

<sup>vi</sup> Johnson, Norman L. and Leone, Fred C. (1964). Statistics and Experimental Design, Vol. 1. John Wiley & Sons, New York.

<sup>vii</sup> Juran, J.M. (1988) Juran’s Quality Control Handbook. 4<sup>th</sup> ed. McGraw-Hill, New York.

<sup>viii</sup> Actually, more generally,  $X - \mu - k$  is used where  $k$  is a constant that can be chosen to provide operating characteristic desired by the designer.

<sup>ix</sup> It is assumed the process was initially in control with a process mean equal to  $\mu$ . If the CUSM signaled after a few samples, then this assumption would be questioned.

<sup>x</sup> See for example: <http://www.itl.nist.gov/div898/handbook/pmc/section3/pmc323.htm>.

<sup>xi</sup> The simulations were run on Statistical Analysis Systems (SAS – release 8.0) using their normal generator: normal(0).